

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4725

Further Pure Mathematics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

- 1 Use the standard results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ to show that, for all positive integers n ,

$$\sum_{r=1}^n (6r^2 + 2r + 1) = n(2n^2 + 4n + 3). \quad [6]$$

- 2 The matrices \mathbf{A} and \mathbf{I} are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.

(i) Find \mathbf{A}^2 and verify that $\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$. [4]

(ii) Hence, or otherwise, show that $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$. [2]

- 3 The complex numbers $2 + 3i$ and $4 - i$ are denoted by z and w respectively. Express each of the following in the form $x + iy$, showing clearly how you obtain your answers.

(i) $z + 5w$, [2]

(ii) z^*w , where z^* is the complex conjugate of z , [3]

(iii) $\frac{1}{w}$. [2]

- 4 Use an algebraic method to find the square roots of the complex number $21 - 20i$. [6]

- 5 (i) Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}. \quad [2]$$

- (ii) Hence find an expression, in terms of n , for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(n+1)(n+2)}. \quad [4]$$

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}$. [1]

- 6 The loci C_1 and C_2 are given by

$$|z - 2i| = 2 \quad \text{and} \quad |z + 1| = |z + i|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]

(ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2 . [2]

7 The matrix \mathbf{B} is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

(i) Given that \mathbf{B} is singular, show that $a = -\frac{2}{3}$. [3]

(ii) Given instead that \mathbf{B} is non-singular, find the inverse matrix \mathbf{B}^{-1} . [4]

(iii) Hence, or otherwise, solve the equations

$$\begin{aligned} -x + y + 3z &= 1, \\ 2x + y - z &= 4, \\ y + 2z &= -1. \end{aligned} \quad [3]$$

8 (a) The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β .

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots α^2 and β^2 . [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots p , $2p$ and $3p$.

(i) Find the value of p . [2]

(ii) Hence find the value of a . [2]

9 (i) Write down the matrix \mathbf{C} which represents a stretch, scale factor 2, in the x -direction. [2]

(ii) The matrix \mathbf{D} is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by \mathbf{D} . [2]

(iii) The matrix \mathbf{M} represents the combined effect of the transformation represented by \mathbf{C} followed by the transformation represented by \mathbf{D} . Show that

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}. \quad [2]$$

(iv) Prove by induction that $\mathbf{M}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$, for all positive integers n . [6]

